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for every plate than to consider whether it will be necessary to do so in particular cases.

In the calculation of the tables from which the diagrams were drawn I have received valuable assistance from Miss Julia Bell and Miss Hilda Gibb, of Girton College, to whom my acknowledgments are due.

We are indebted to the Government Grant Fund of the Royal Society for a grant to secure their assistance in the reduction of the *Eros* photographs.

Cambridge Observatory: 1903 *January* 3.

Note on the Use of Mr. Aldis' Tables of the Function $\frac{1}{2}(\theta + \cos\theta)$ *in Determining the Elements of an Orbit.* By H. C. Plummer, M.A.

1. In the second part of his explanation* of the use of tables of $\frac{1}{2}(\theta + \cos\theta)$ Mr. Aldis has shown that they can be applied to the problem of finding the elements of an elliptic orbit when two focal distances, the angle between them, and the time occupied in passing from one position to the other are known. An inspection of the example which he has worked out makes it evident that the practical utility of the method may be greatly enhanced if a convenient way can be devised of finding preliminary approximations to the values of the quantities required. It is the object of this note to supply the want. A graphical method will be first described; but this is to be regarded chiefly as the basis of the analytical method of approximation which follows.

2. By the use of Lambert's theorem the problem is reduced to the solution for a, ϵ, and δ of this system of three equations:

$$\frac{kt}{a^{\frac{3}{2}}} = (\epsilon - \sin\epsilon) - (\delta - \sin\delta) \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$1 - \cos\delta = \frac{r + r' - c}{2a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

$$1 - \cos\epsilon = \frac{r + r' + c}{2a} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

The notation is that of Mr. Aldis' paper. When a is known the eccentricity e can be easily deduced. It will be convenient to write

$$\sigma_1^2 = r + r' + c\,; \quad \sigma_2^2 = r + r' - c.$$

Now if

$$x_1 = \delta - \sin\delta\,; \quad y_1 = 1 - \cos\delta$$
$$x_2 = \epsilon - \sin\epsilon\,; \quad y_2 = 1 - \cos\epsilon$$

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* *Monthly Notices*, vol. lxii. p. 638.

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x_1, y_1 and x_2, y_2 are the coordinates of points lying on the cycloid whose equations are

$$x = \theta - \sin\theta \; ; \; y = 1 - \cos\theta$$

The coordinates are also connected by the relations

$$kt = a^{\frac{3}{2}}(x_2 - x_1)$$

$$2ay_1 = \sigma_2{}^2 \; ; \; 2ay_2 = \sigma_1{}^2$$

If a be eliminated we have

$$2\sqrt{2kt}\,y_1{}^{\frac{3}{2}} = \sigma_2{}^3(x_2 - x_1)$$

$$2\sqrt{2kt}\,y_2{}^{\frac{3}{2}} = \sigma_1{}^3(x_2 - x_1)$$

3. These equations have a geometrical interpretation. Let p and q (fig. 1) be the points (x_1, y_1) and (x_2, y_2) on the cycloid

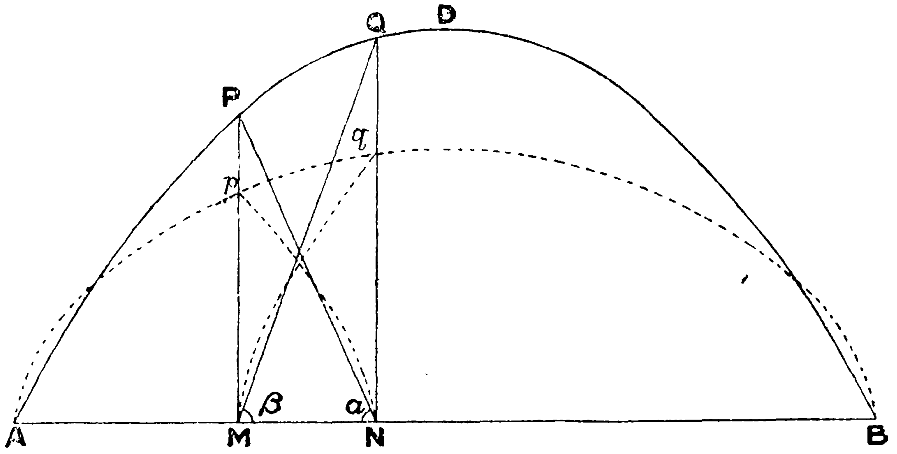

Fig. 1.

ACB. The meaning of the equations is that semi-cubical parabolas with known parameters and axes parallel to $x = 0$ will pass respectively through p and q when their vertices are at N and M, the feet of the ordinates at q and p. It then becomes evident that the figure will be greatly simplified by a transformation which corresponds to the substitution

$$x = \mathrm{X} \; ; \; y^{\frac{3}{2}} = \mathrm{Y}$$

i.e. by raising each ordinate to the power $\frac{3}{2}$. The derived curve ADB (fig. 1) which replaces the cycloid has for its equations

$$\mathrm{X} = \theta - \sin\theta \; ; \; \mathrm{Y} = (1 - \cos\theta)^{\frac{3}{2}} = 2\sqrt{2}\sin^3 \tfrac{1}{2}\theta \quad \ldots \quad (4)$$

The semi-cubical parabolas are replaced by the straight lines PN, MQ, inclined to AB at angles α, β, such that

$$\tan\alpha = \sigma_2{}^3/2\sqrt{2kt} \; ; \; \tan\beta = \sigma_1{}^3/2\sqrt{2kt} \quad \ldots \quad (5)$$

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4. A graphical method of finding the required approximations is thus suggested. The curve ADB given by (4) must first be constructed on squared paper, and the base line AB may conveniently be graduated in θ. Then it is necessary to place the lines PN, QM at known inclinations to the base in such a manner that their intersections with the curve project (as in fig. 1) on to their intersections with the base. A convenient way of doing this is to draw the lines PN, QM on ground glass, with a third line in the direction of the base, and to move the figure so obtained over the paper until as good a fit as possible is obtained. Rough values of ϵ and δ can then be read off. But actual trial of the method in one or two cases has seemed to show that it is not susceptible of any great accuracy. This is in a great measure due to the fact that the given interval of time is in general small. The analytical method of approximation now to be described takes advantage of this fact, and will become the more effective as the graphical method becomes more impracticable.

5. In fig. 2, which corresponds to the essential part of fig. 1,

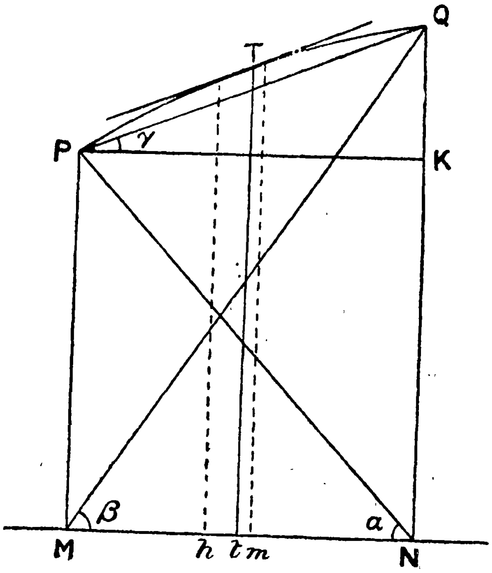

Fig. 2

let PK be drawn parallel to MN, and let the angle QPK $= \gamma$. This angle is easily determined, for

$$\tan\gamma = \tan\beta - \tan\alpha$$

Let T be the point of contact of that tangent to the curve which is parallel to PQ, and let $\theta = \tau$ at this point. It is easily found that

$$\tan\gamma = \frac{3}{\sqrt{2}}\cos\tfrac{1}{2}\tau$$

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Hence by (5)

$$\cos \tfrac{1}{2} \tau = (\sigma_1{}^3 - \sigma_2{}^3)/6kt \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

a result which presents a curious analogy to Euler's theorem. In fact, the latter is hereby deduced from Lambert's theorem * if it is noticed that, in the limiting case of the parabola, $\delta = \varepsilon = \tau = 0$.

6. It will be convenient to put

$$2\eta = \varepsilon + \delta\,;\ 2\zeta = \varepsilon - \delta \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Now τ must clearly lie between δ and ϵ, and is indeed a good first aproximation to the value of η. Let ζ_1 be the corresponding value of ζ which makes (2) and (3) consistent. But these equations give

$$\sigma_1 \sin \tfrac{1}{2} \delta = \sigma_2 \sin \tfrac{1}{2} \epsilon$$

which can be written in the modified form

$$\tan \tfrac{1}{2}\zeta = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} \tan \tfrac{1}{2}\eta \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

Hence

$$\tan \tfrac{1}{2}\zeta_1 = \frac{1-n}{1+n} \tan \tfrac{1}{2}\tau \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

where $n = \sigma_2/\sigma_1$. Thus τ and ζ_1 are found very easily by (6) and (9), and the corresponding approximations to δ and ε are deduced immediately.

7. It is now easy to see how the approximation can be improved by a simple step. Let (fig. 2) t be the foot of the ordinate through T, m the foot of the ordinate midway between PM and QN, and h the foot of the ordinate through the point for which $\theta = \tfrac{1}{2}(\varepsilon + \delta)$. Let $y = \phi(x)$ be the equation of the curve and x and $x+h$ the abscissæ of P and Q. Then by a well-known theorem

$$\frac{\phi(x+h) - \phi(x)}{h} = \phi'(x + fh)$$

where

$$f = \tfrac{1}{2} + \tfrac{1}{24} \cdot \frac{\phi'''(x)}{\phi''(x)} h$$

if the expansion of f is not carried beyond the first power of x. Hence

$$tm = -\tfrac{1}{24} \cdot \frac{\phi'''(x)}{\phi''(x)} h^2$$

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* Adams has given a simple proof that Euler's theorem can be deduced as a limiting case.—*Collected Scientific Papers*, vol. i. p. 413.

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And if $x = \psi(\theta)$ and θ becomes $\theta+\Delta\theta$ at Q, then

$$\begin{aligned} km &= \tfrac{1}{2}\psi(\theta)+\tfrac{1}{2}\psi(\theta+\Delta\theta)-\psi(\theta+\tfrac{1}{2}\Delta\theta) \\ &= \tfrac{1}{8}\psi''(\theta)\,.\,\Delta\theta^2 \\ \therefore\ ht &= \left\{\tfrac{1}{8}\psi''(\theta)+\tfrac{1}{24}\,.\,\frac{\phi'''(x)}{\phi''(x)}\,[\psi'(\theta)]^2\right\}\Delta\theta^2 \\ &= +\tfrac{5}{48}\sin\theta\,.\,\Delta\theta^2 \end{aligned}$$

when the differentiations are performed. The correction to be applied to τ in order to give a second approximation to the value of η is therefore

$$-ht \div \psi'(\theta) = -\tfrac{5}{48}\cot\tfrac{1}{2}\theta\,.\,\Delta\theta^2$$

If, as above, the calculation is made in terms of the lower end of the chord, $\theta = \delta$; and starting from the upper end we should find $\theta = \epsilon$. It is therefore natural to adopt $\theta = \tau$. Also $\Delta\theta = 2\zeta$. Hence it is inferred that if

$$\Delta\tau = -\tfrac{5}{12}\cot\tfrac{1}{2}\tau\,.\,\zeta^2 \qquad \dots \quad \dots \quad \dots \quad (10)$$

then $\tau+\Delta\tau$ is the second approximation to the value of η, from which improved values of ϵ and δ can be deduced in the manner already explained. All the angles occurring in the foregoing formulæ and the constant k are supposed to be expressed in circular measure.

8. The result of practical trials is to show that the method is extremely effective. In the first instance the example which was taken by Mr. Aldis from the *Theoria Motus* was tried. This gave

$$\begin{aligned} \tau &= 78^\circ\ 38'\ 21''\cdot6\ ;\quad \zeta_1 = 3^\circ\ 8'\ 21''\cdot6\ ;\quad \epsilon_1 = 81^\circ\ 46'\ 43''\cdot2 \\ -\Delta\tau &= 5\ \ 14\cdot8\ ;\quad \zeta_2 = 3\ \ 8\ \ \ 4\cdot0\ ;\quad \epsilon_2 = 81\ \ 41\ \ 10\cdot8 \end{aligned}$$

Now Mr. Aldis finds for the correct value of ϵ, 81° 41′ 11″·5, and hence for log a, 0·4224385, while the value given by Gauss for log a is 0·4224389. The above value of ϵ_2 gives log a = 0·4224400. And if $\Delta\tau$ is recalculated by means of ζ_2 in the place of ζ_1, the correct value of ϵ is obtained. A second trial was made with an artificial example given by Dr. Moulton * in which the eccentricity was unusually large. In this case the value assigned to a was 2·65. The logarithms of three radii vectores are given only to six figures. The value found for ϵ_2 from the shorter period of thirty days gives $a = 2{\cdot}650063$, which shows a fair agreement with the value actually assigned. But if the extreme places of the planet, with an interval of sixty-five days, are taken, the case is by no means so good. From the value found for ϵ_2, $a = 2{\cdot}6519$. It is true that an improvement can be made by recalculating $\Delta\tau$ as mentioned above, but the

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* *Astr. Journal*, No. 510.

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method would lose the advantage of its simplicity thereby. It is better to treat ϵ_2 as an approximation which facilitates the use of Mr. Aldis' tables. Cases in which the interval of time is so long must occur very rarely, if at all, in the practical applications of the problem. But in such cases, and whenever the highest degree of accuracy is required, the value of the tables, as far as this problem is concerned, will consist in giving the final improvement, or at least the verification, to approximate solutions found by some preliminary method.

9. It is, however, desirable to examine the nature of the second approximation more closely. The tangent to the curve (4) at the point $\theta = \tau$ is parallel to the chord joining the points $\theta = \eta - \zeta$ and $\theta = \eta + \zeta$. Therefore

$$\tfrac{3}{4}\cos\tfrac{1}{2}\tau = \frac{\sin^3\frac{1}{2}(\eta+\zeta) - \sin^3\frac{1}{2}(\eta-\zeta)}{2\zeta - \sin(\eta+\zeta) + \sin(\eta-\zeta)}$$

$$\therefore 3\cos\tfrac{1}{2}\tau = \frac{3\cos\frac{1}{2}\eta\sin\frac{1}{2}\zeta - \cos\frac{3}{2}\eta\sin\frac{3}{2}\zeta}{\zeta - \cos\eta\sin\zeta}$$

$$\therefore \frac{3\cos\frac{1}{2}\tau}{2\cos\frac{1}{2}\eta}$$

$$= \frac{\frac{1}{2}\zeta(3c_1 - 3c_3) - \frac{1}{6.8}\zeta^3(3c_1 - 27c_3) + \frac{1}{120.32}\zeta^5(3c_1 - 243c_3) - \dots}{\zeta(c_1 - c_3) + \frac{1}{6}\zeta^3(c_1 + c_3) - \frac{1}{120}\zeta^5(c_1 + c_3) + \dots}$$

where

$$c_1 = \cos\tfrac{1}{2}\eta\,; \quad c_3 = \cos\tfrac{3}{2}\eta$$

The expansion in ζ is clearly an even function, and if terms of the order ζ^6 be neglected

$$\frac{\cos\frac{1}{2}\tau}{\cos\frac{1}{2}\eta} = \frac{(c_1 - c_3) - \frac{1}{6.4}\zeta^2(c_1 - 9c_3) + \frac{1}{120.16}\zeta^4(c_1 - 81c_3)}{(c_1 - c_3) + \frac{1}{6}\zeta^2(c_1 + c_3) - \frac{1}{120}\zeta^4(c_1 + c_3)}$$

$$\therefore \frac{\cos\frac{1}{2}\eta - \cos\frac{1}{2}\tau}{\cos\frac{1}{2}\tau} = \frac{\frac{5}{24}\zeta^2(c_1 - c_3) - \frac{1}{1920}\zeta^4(17c_1 - 65c_3)}{(c_1 - c_3) - \frac{1}{24}\zeta^2(c_1 - 9c_3)}$$

$$= \tfrac{5}{24}\zeta^2 - \tfrac{1}{5760}\cdot\frac{c_1 + 255c_3}{c_1 - c_3}\cdot\zeta^4$$

$$= \tfrac{5}{24}\zeta^2 + \tfrac{1}{5760}(191 - 64\cot^2\tfrac{1}{2}\eta)\zeta^4$$

10. In the coefficient of ζ^4, τ may be written for η. Then

$$\sec\tfrac{1}{2}\tau\Delta\cos\tfrac{1}{2}\tau = \tfrac{5}{24}\zeta^2 + \tfrac{1}{5760}(191 - 64\cot^2\tfrac{1}{2}\tau)\zeta^4$$

where $\Delta\cos\frac{1}{2}\tau$ is the correction to be added to $\cos\frac{1}{2}\tau$ to obtain $\cos\frac{1}{2}\eta$, so that

$$\Delta\cos\tfrac{1}{2}\tau = -\tfrac{1}{2}\sin\tfrac{1}{2}\tau\,.\,\Delta\tau - \tfrac{1}{8}\cos\tfrac{1}{2}\tau(\Delta\tau)^2$$

Hence, to the degree of approximation adopted,

$$\Delta\tau = -\tfrac{5}{12}\cot\tfrac{1}{2}\tau\,.\,\zeta^2 - \tfrac{1}{2880}\cot\tfrac{1}{2}\tau(191 + 61\cot^2\tfrac{1}{2}\tau)\zeta^4$$

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which confirms the formula (10) previously obtained and gives a further development. Now since

$$4\tan^2\tfrac{1}{2}\zeta=\zeta^2+\tfrac{1}{6}\zeta^4$$

it is possible to write

$$\Delta\tau=-\tfrac{5}{3}\cot\tfrac{1}{2}\tau\tan^2\tfrac{1}{2}\zeta-\tfrac{1}{2880}\cot\tfrac{1}{2}\tau(61\cot^2\tfrac{1}{2}\tau-9)\zeta^4\ldots\quad(11)$$

Therefore if powers of ζ above the third be neglected it is more accurate to write (10) in the form

$$\Delta\tau=-\tfrac{5}{3}\cot\tfrac{1}{2}\tau\tan^2\tfrac{1}{2}\zeta$$

which again, by means of (9), may be expressed

$$\Delta\tau=-\tfrac{5}{3}\left(\frac{1-n}{1+n}\right)^2\tan\tfrac{1}{2}\tau\quad\ldots\quad\ldots\quad\ldots\quad(12)$$

Thus the second approximation to η may be derived directly from (6) and (12) before an approximation to ζ has been calculated. The work to this stage will therefore be very rapid, and it will be easy to judge from the value then found for ζ whether it is worth while to refine on the approximation by means of (11).

11. The representation of the consequences of Lambert's theorem by means of a curve related to the cycloid was suggested by the method I have previously described * for finding graphically an approximation to the solution of Kepler's equation by means of the trochoid. The application of Mr. Aldis' tables to the latter problem will be rendered more advantageous by using them as an auxiliary to some such approximate method as the one to which I refer. My description of that method was not, as it soon appeared, original. It had, for instance, been published by Dr. Rambaut † a few years earlier; and quite lately it has again been brought forward by B. Gonggrijp. ‡ But the principle of the method is as old as Wallis and Newton.

Note added 1903 *Jan.* 24.

The purpose of the foregoing note was, as explained in the beginning, merely to supply an approximate method which might serve to facilitate the use of Mr. Aldis' tables, and the examples mentioned in § 8 suffice to show that this object has been fully attained. But the method as further developed in §§ 9 and 10 is something more than a mere auxiliary to the tables. It provides an adequate and convenient means of solving Lambert's equation in cases of moderate difficulty without the aid of any special tables.

The essence of the method is contained in the equations (6),

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* *Monthly Notices*, vol. lvi. (1896), p. 317.

† *Ibid.* vol. l. (1890), p. 301.

‡ *A. N.* (1901), No. 3720.

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(8) and (11), by means of which the values of η and ζ are found by successive approximations. It is well to remark that

$$\zeta = \tfrac{1}{2}(E'-E)\,;\quad \cos\eta = e\cos\tfrac{1}{2}(E'+E)$$

where E, E′ are the eccentric anomalies of the planet; and that

$$ap = rr'\sin^2\Delta\,\mathrm{cosec}^2\zeta$$

where 2Δ is the difference of the true anomalies. This explains the nature of the expansion (11), which might, if necessary, be developed further without serious difficulty. Its advantage consists in its rapid convergence for moderate values of ζ and in the fact that to the order ζ^5 it consists of only two terms, of which the second is very small. For actual calculation (11) may be written in the form

$$-\tfrac{1}{2}\Delta\tau = Z + FZ^2 \quad \dots \quad \dots \quad (11')$$

where

$$Z = \tfrac{5}{6}\cot\tfrac{1}{2}\tau\tan^2\tfrac{1}{2}\zeta$$

$$F = \tfrac{1}{250}(61-9\tan^2\tfrac{1}{2}\tau)\cot\tfrac{1}{2}\tau$$

When log Z is found, Z can be taken from a table of logarithmic sines; and the following table, which will apply to eccentricities of at least 0·5, gives log F in such a way that the term FZ^2 will be found in seconds of arc:—

Table of log F.

$\frac{1}{2}\tau$.	log F.	Diff.	$\frac{1}{2}\tau$.	log F.	Diff.	$\frac{1}{2}\tau$.	log F.	Diff.
30°	4·9186	−192	40°	4·7305	−190	50°	4·5238	−240
31	·8994	190	41	·7115	193	51	·4998	250
32	·8804	189	2	·6922	195	52	·4748	262
33	·8615	187	43	·6727	198	53	·4486	276
34	·8428	187	44	·6529	202	54	·4210	291
35	·8241	186	45	·6327	206	55	·3919	310
36	·8055	187	46	·6121	212	56	·3609	332
37	·7868	186	47	·5909	217	57	·3277	358
38	·7682	188	48	·5692	223	58	·2919	390
39	4·7494	−189	49	4·5469	−231	59	·2529	−430
						60	4·2099	

The more complete formulæ have been applied to the example already mentioned in § 8, in which the interval of time is 65 days and the eccentricity 0·3. The result is quite satisfactory, and shows that the formulæ are as accurate as the data require. As the second interval between the observations does not enter into this part of the computation, and is therefore unlimited, this example is a far more severe test than when two intervals of 30 and 35 days are used. The additional term FZ^2 amounts

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to $4''\cdot77$. It may be necessary, as it is in this case, to repeat the calculation by means of (8) and (11′) two or three times, but the process is in practice very simple and rapid. It is convenient to use logarithmic tables giving every second of arc (*e.g.* Bagay's), to keep the openings for $\frac{1}{2}\eta$, $\frac{1}{2}\zeta$, and $\frac{1}{2}\Delta\tau$ marked, and instead of repeating the whole of the figures (which in any case are very few) it is sufficient merely to write the *corrections* in parallel columns. In fact, when σ_1 and σ_2 are known it takes very little time to find η and ζ, and hence a, p and e. The method is summarised in the following scheme :—

Summary of Calculation.

(1) Calculate σ_1, σ_2 by means of

$$\sigma_1{}^2=r+r'+c\ ;\ \sigma_2{}^2=r+r'-c$$

(2) Calculate $n=\sigma_2/\sigma_1$, and hence $\log \mathrm{N}=\log(1-n) -\log(1+n)$

(3) Find $\frac{1}{2}\tau$ by means of

$$\cos\tfrac{1}{2}\tau=(\sigma_1{}^3-\sigma_2{}^3)/6kt$$

(4) Find $\frac{1}{2}\Delta\tau$ from

$$\tfrac{1}{2}\Delta\tau=-\tfrac{5}{6}\mathrm{N}^2\tan\tfrac{1}{2}\tau$$

(5) Hence $\frac{1}{2}\eta_2=\frac{1}{2}(\tau+\Delta\tau)$, and ζ_2 can be found from

$$\tan\tfrac{1}{2}\zeta_2=\mathrm{N}\tan\tfrac{1}{2}\eta_2$$

(6) With ζ_2 as an approximate value of ζ find log Z from

$$\mathrm{Z}=\tfrac{5}{6}\cot\tfrac{1}{2}\tau\tan^2\tfrac{1}{2}\zeta$$

and recalculate $\frac{1}{2}\Delta\tau$ by means of

$$-\tfrac{1}{2}\Delta\tau=\mathrm{Z}+\mathrm{FZ}^2$$

(7) If necessary repeat the operations (5) and (6) until the equations

$$-\tfrac{1}{2}(\eta-\tau)=-\tfrac{1}{2}\Delta\tau=\mathrm{Z}+\mathrm{FZ}^2$$

$$\tan\tfrac{1}{2}\zeta=\mathrm{N}\tan\tfrac{1}{2}\eta$$

are both satisfied.

(8) When η and ζ are thus known with sufficient accuracy a is found from

$$a=\sigma_1{}^2/4\sin^2\tfrac{1}{2}(\eta+\zeta)$$

and p from

$$ap=rr'\sin^2\Delta\operatorname{cosec}^2\zeta$$

Finally the eccentricity is found from a and p.

University Observatory, Oxford :
1903 *Jan.* 8.

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On a New and Accurate Method of Determining Time, Latitude, and Azimuth with a Theodolite. By W. E. Cooke, M.A., Government Astronomer, Western Australia.

The *principle* of this method is not new. It was, I believe, first advocated by Mr. S. C. Chandler, who designed an instrument which he called an "Almucantar," developed the mathematical theory, and made a series of very fine observations at the Harvard College Observatory. An instrument of that kind, however, does not form part of a surveyor's ordinary outfit, whilst a theodolite does; and my present purpose is to show how the method may be applied to this universal instrument, and how by its means results of far greater accuracy than those usually obtained may be deduced. Briefly, the spirit level can be used instead of a mercury flotation in order to ensure constancy in zenith distance, and this brings the method within the power of every surveyor.

I shall first show it in connection with a 5-inch theodolite, and state the results of a few observations already made. Afterwards I propose to show that with a 12-inch instrument results can be obtained which, I believe, exceed in accuracy anything in field work hitherto published.

Five-inch Theodolite.

The observation for time and latitude consists in observing the time of transit of *Nautical Almanac* stars over the horizontal thread when the instrument is set so as to sweep a small circle in the sky parallel with the horizon, and at an altitude *about* equal to the observer's latitude. For azimuth it will be necessary, in addition, to take the time of transit across the vertical thread and to read the azimuth circle. An ordinary watch which possesses a seconds hand will do for timekeeper. The advantages may as well be enumerated here.

For time and latitude errors of vernier reading do not exist, as we require the circles for approximate setting purposes only. Extreme accuracy of construction and adjustment is unnecessary. We require only ordinary care in levelling. Error of collimation is absolutely immaterial. We do not even require that the bubbles shall be properly adjusted. All we require in the way of adjustment is that the vertical axis shall be reasonably vertical, so that the cross bubble on the altitude circle shall remain *fairly* steady as the instrument is swung round in azimuth, and great accuracy is not required even for this. As a matter of fact the instrument I used had been standing in a corner for months, and I simply took it out and levelled it up.

Yet, notwithstanding this apparent want of respect, results of considerable accuracy may be easily obtained. Those obtained in two evenings' trial will be stated almost immediately.